Formal Logic Additive Inference Drill—Expanded Explanations

Following are the expanded explanations for the Formal Logic Additive Inference Drill that appears in Chapter 13 of the PowerScore LSAT Logical Reasoning Bible.

1. Some As are Bs.
   No Bs are Cs.
   All Cs are Ds.

   Diagram: A ←s→ B ←|→ C ⟶ D

   Inferences: A ←s→ ~~C~~
   D ←s→ ~~B~~

The diagram for the three statements in this problem lays itself out sequentially from left to right, making this fairly easy to diagram. Using the double-not arrow between B and C (B ←|→ C) is critical because it allows all four variables to be connected in a single, cohesive diagram.

The first inference is derived from the classic connection between A, B, and C. To make the inference, start at A, since A is at one end of the chain and is involved in a *some* relationship. From A we can ride over to B. Once we arrive at B, is there a "track" leading away? Yes, there is an outgoing track to C, and thus we can travel from A to C. The weakest relationship along the path is "*some,* and there is relevant negativity in the form of the negative between B and C, so the inference is "A some not C," as diagrammed above. Note that the "not" is placed on C because the negative relationship is between B and C (and thus the negative is not related to A).

The second inference begins at the other end of the chain, with D. To make this inference, you must "go backwards" across the arrow, which can be done because *some* is inherently present in the C ⟶ D relationship. So, from D we can ride over to C. Once we arrive at C, is there a "track" leading away? Yes, there is an outgoing track to B, and thus we can travel from D to B. The weakest relationship along the path is *some,* and there is relevant negativity in the form of the negative between C and B, so the inference is "D some not B," as diagrammed above.

With inferences present between A and C, and B and D, the two "interior" variables in the chain (B and C) have been connected to the variable furthest from each. Because the relationships between A and B, B and C, and C and D would only result in inherent inferences, we won't catalogue those. Thus, the one remaining relationship to check is the one between the two end points—A and D. While it may initially appear that an inference can be drawn between the two, it cannot because the double-not arrow between B and C creates an issue. If you were to start with A, you can travel to C, with a resulting A ←s→ ~~C~~ inference. However, there is no way to connect that to D, because the relationship between C and D is C ⟶ D. Basically, A connects to C, and so to connect to D you would need C ⟶ D.

2. All Xs are Ys.
Some Ys are Zs.
Most Xs are Ws.

Diagram: W ←(M)— X ⟶ Y ←(S)→ Z

Inference: W ←(S)→ Y

The diagram for these three statements is more challenging than the diagram for #1. The first statement, "All Xs are Ys," is a simple single-arrow statement. The second statement easily links to that one via Y. The third statement can initially present some problems because it only connects to X, which is the first term in the chain at this point. Thus, to connect to X, the *most* arrow needs to point from right to left. Combining the three statements in this fashion results in the diagram above.

To make inferences, start with each end of the chain. Starting with W, you can move to X via the inherent *some* inference between W and X. Once at X, you can follow the *all* arrow to Y. The weakest relationship along the path is *some,* and there is no relevant negativity, so the inference is "W some Y," as diagrammed above.

Moving to the other end of the chain, we begin with Z. Since the relationship with Y is *some*, we can then move to Y. However, there is only an incoming arrow, and so you cannot move over to X. Hence, no inference can be drawn.

With no inference available between X and Z, there automatically can be no inference between W and Z. In other words, if there is no link between three consecutive variables along the chain—such as with X, Y, and Z in this example—then there can be no link with a variable linked to X (such as W here).

3. No Es are Fs.
All Fs are Gs.
All Gs are Hs.

Diagram: E ←+→ F ⟶ G ⟶ H

Inferences:
G ←(S)→ ~~E~~
F ⟶ H
H ←(S)→ ~~E~~

The diagram for the three statements in this problem lays itself out sequentially from left to right in a single, cohesive diagram, providing you use the double-not arrow between E and F (E ←+→ F). Using the double-not arrow allows for a positive F, which can then be linked to G via F ⟶ G.

Normally, E would be a fruitful point to start our analysis, but in this case E is connected to F with a double-not arrow, and the 2nd Principle of Making Formal Logic Inferences states that, "When making inferences, do *not* start with a variable involved in a double-not arrow relationship and then try to "go across" the double-not arrow." Consequently, we will abandon E and begin at the other end of the chain, with H.

However, this is more troublesome than usual. While we can go "backwards" from H to G via the inherent inference *some*, the same relationship exists from G back to F, and two *somes* in a row never yield an inference. So, starting at either end of the chain isn't helpful—which is annoying! Instead, let's look at the two "interior" variables.

Beginning at F, you can see two *all* arrows heading to the right: F ⟶ G ⟶ H. The combination of these two arrows produces the inference F ⟶ H. This brings up an interesting point: while you can't draw an inference starting at H in that FGH chain, you can draw an inference when you start at F. And once you have that inference, it's set in stone. Thus, not only do we know F ⟶ H, we also know that from the inherent inferences that H ←s→ F. We'll return to this bit of information in a moment.

Looking at the other interior variable—G—can we draw any inferences? Yes, starting at G you can move to F via the inherent *some*, and then use the negative arrow over to E. The weakest relationship along the path is *some,* and there is relevant negativity, so the inference is "G some not E," as diagrammed above.

At this point it would be easy to walk away and think you have made all of the available additive inferences. However, always remember to mentally recycle your inferences back into the chain when there are four or more variables involved. Recycling our two prior inferences leads to the following two diagrams:

E ←+→ F ⟶ H (recycles F ⟶ H)

~~E~~ ←s→ G ⟶ H (recycles G ←s→ ~~E~~)

If you analyze each chain, you may realize that each results in the same inference: H ←s→ ~~E~~. In other words, recycling either inference in an attempt to connect the far ends of the chain (E and H) ultimately results in the same inference (which makes sense because the inference between E and H is constant).

The last point is a key one, because often the toughest Formal Logic problems require you to recycle inferences to draw a "super inference" that stretches the length of the chain. That can be quite difficult unless you've practiced it beforehand.

4. Some T are Us.
All Us are Vs.
All Ts are Ss.

Diagram: S ⟵ T ⟷(s) U ⟶ V

Inferences:
T ⟷(s) V
U ⟷(s) S
S ⟷(s) V

In this diagram, the first statement—"Some T are Us"—is at the center of the diagram, and *all* arrows lead away to the two endpoints (S and V).

As it turns out, starting at either endpoint will result in failure: although the first step will be to move to the inside variable via the inherent *some* (S to T, or V to U), no next step can be taken because of the presence of a second *some* (which is between T and U). Consequently, you must look to the interior variables (T and U) as starting points.

If you start at T, you can move to U via *some*. Once at U, you can follow the *all* arrow to V. The weakest relationship along the path is *some,* and there is no relevant negativity, so the inference is "T some V," as diagrammed above.

If you start at U, you can move to T via *some*. Once at T, you can follow the *all* arrow to S. The weakest relationship along the path is *some,* and there is no relevant negativity, so the inference is "U some S," as diagrammed above.

At this point, remember to mentally recycle your inferences back into the chain. Recycling our two inferences leads to the following two diagrams:

S ⟵ T ⟷(s) V (recycles T ⟷(s) V)

S ⟷(s) U ⟶ V (recycles U ⟷(s) S)

If you analyze each chain, you can see that each results in the same inference: S ⟷(s) V.
In other words, recycling either inference in an attempt to connect the far ends of the chain (S and V) ultimately results in the same inference.

Note: The form of this problem— S ⟵ T ⟷(s) U ⟶ V —has appeared in several LSAT questions and the final S ⟷(s) V inference has always been the correct answer.

5. Most Is are Js.
All Js are Ks, and all Ks are Js.
All Ks are Ls.

Diagram: $I \xrightarrow{M} J \longleftrightarrow K \longrightarrow L$

Inferences:
$I \xrightarrow{M} K$
$J \longrightarrow L$
$I \xrightarrow{M} L$

The diagram for the three statements in this problem lays itself out sequentially from left to right, making this fairly easy to diagram. Notably, for the first time in this drill, there is the appearance of a *most* statement, as well as the appearance of a full double arrow statement (between J and K). Double arrows are extremely powerful, and the presence of the double-arrow creates more forceful inferences than usual.

Starting at I, the *most* arrow can be followed over to J. Once at J, there is an *all* arrow to K, and so an inference can be drawn between I and K. The weakest relationship along the path is *most,* and there is no relevant negativity, so the inference is "I most K," as diagrammed above. And, since there is another *all* arrow leading from K to L, that arrow can be followed as well (which is the same as recycling the inference), resulting in the inference that "I most L," also diagrammed above.

In addition, in the prior paragraph we noted that there are basically two *all* arrows between J and L: $J \longleftrightarrow K \longrightarrow L$. Consequently, we can ride from J to K, and then from K to L, resulting in the inference that $J \longrightarrow L$.

At this point, we haven't started an analysis from the other endpoint, L, yet. If you start at L, you can move to K via the inherent *some* inference between K and L. Once at K, you can follow the *all* arrow to J. The weakest relationship along the path is *some,* and there is no relevant negativity, so the inference is "L some J." But wait, what's wrong—that is not diagrammed above! Or is it?

Consider that we already determined that $J \longrightarrow L$. Inherent in the $J \longrightarrow L$ relationship is that "some Js are Ls" and that "some Ls are Js." That last point matches the "inference" we made in the prior paragraph, and so the reason that "L some J" is not shown in our inference list is because it is already inherently contained within $J \longrightarrow L$. A tricky point for sure, and exactly the kind of point that could be exploited by LSAC in a tough question.

6. Some Ns are Os.
   No Os are Ps.
   No Ps are Qs.
   All Qs are Rs.

   Diagram: $N \overset{s}{\longleftrightarrow} O \longleftrightarrow\!\!\!\!\!\!\!\!+\;\;\;\; P \longleftrightarrow\!\!\!\!\!\!\!\!+\;\;\;\; Q \longrightarrow R$

   Inferences: $N \overset{s}{\longleftrightarrow} \not{P}$
   $R \overset{s}{\longleftrightarrow} \not{P}$

The diagram for the four statements in this problem lays itself out sequentially from left to right in a single, cohesive diagram.

While a diagram with five variables would seem likely to offer up a slew of inferences, the presence of consecutive double-not arrows serves to limit the number of available inferences (because two consecutive double-not arrows form a "wall" of sorts that stops all inferences paths). In other words, the $O \longleftrightarrow\!\!\!\!\!\!\!\!+\;\;\;\; P \longleftrightarrow\!\!\!\!\!\!\!\!+\;\;\;\; Q$ relationship doesn't allow for any inferences between O and Q, and thus variables linking to O or Q can't get past P to make a further inference.

The above point notwithstanding, there are still a few inferences to be made. Starting at N, we can ride the *some* over to O, and from O we can use the arrow to ride over to P. The weakest relationship along the path is *some,* and there is relevant negativity on P, so the inference is "N some not P," as diagrammed above.

At the other end of the chain, let's start at R and see what happens. From R, we can ride over to Q using the inherent *some*. Once at Q, there is a negative arrow over to P. Thus, we can make an inference between R and P. The weakest relationship along the path is *some,* and there is relevant negativity on P, so the inference is "R some not P," as diagrammed above.

As far as the interior variables, we have already documented that we cannot draw an inference between O and Q, which renders them useless as far as inferences other than the ones already drawn. P, the "center" variable is connected to N and R in separate inferences, and we can recycle those to create the following chain:

$$N \overset{s}{\longleftrightarrow} \not{P} \overset{s}{\longleftrightarrow} R$$

However, this chain does not yield an inference because two consecutive *somes* are not strong enough to allow for any deductions. Consequently, the two inferences above are the only additive inferences that can be drawn from this chain.