Negative Sufficient Conditions—An Expanded Discussion

Conditional statements appear frequently on the LSAT, and most students eventually become very comfortable with identifying and understanding conditional statements. The majority of conditional statements contain what could be called a "positive" or non-negated sufficient condition—one where something occurs as opposed to not occurring. Consider the following statement:

When the buzzer sounds, the game ends.

This sentence features the sufficient indicator "when," and thus "the buzzer sounds" is the sufficient condition, and the necessary condition is then "the game ends." This type of statement is fairly simple for the typical student to understand, and most students wouldn't need or use a diagram to help understand this relationship (which is in fact the case for most conditional statements, especially the ones in the Logical Reasoning section).

Part of what makes this relationship easy to handle is that the sufficient condition involves something actually occurring—the sound of a buzzer. It's not difficult to mentally picture that sound occurring, and then relating that to the end of the game. "Positive" sufficient conditions—which involve acts of occurrence—are often easy to mentally mark, and generally easier to handle.

In a Logic Game, for example, you might come across the following rule:

If P sings third, then Q must sing fourth.

Diagram: $P_3 \longrightarrow Q_4$

Once you've assessed this rule, you can mentally track the instances where P sings third, and whenever that outcome occurs, you can enact the rule. In other words, you simply have to wait for a single occurrence in order to manage this rule.
Compare this to the same rule, with the small change of making the sufficient condition negative:

If P does *not* sing third, then Q must sing fourth.

Diagram: $P_3 \longrightarrow Q_4$

Fundamentally, this rule is very similar to the first one above: both feature two variables in relation to two specific spots, and they are involved in a single arrow relationship. However, the negative on the sufficient condition dramatically changes the way the second rule works.

In this second rule, any time P *isn't* third, then the rule activates. On the surface, this seems like it shouldn't be difficult to remember, because now the rule has many more possible situations when it comes into play. For example, if this was a game with seven total slots, then if P was in any of the other spaces (besides third), then the rule would be enacted. However, a problem occurs because this rule is based on the non-occurrence of an event: it only applies when P is *not* third. That presentation is harder for most people to keep in mind because instead of being able to mark the rule as occurring in relation to a specific event, it's now marked by a *lack* of occurrence. Our minds are better programmed to identify and react to occurrences as opposed to non-occurrences, hence the special challenge posed by these types of statements.

In one sense, it would be easier of the test makers posed the rule by stating what actually occurs (and we'll use our seven slots example discussed in the prior paragraph):

> If P does *not* sing third, then Q must sing fourth.
>
> Diagram: $P_{1/2/4/5/6/7} \longrightarrow Q_4$
>
> Note: P singing fourth is impossible under this reinterpretation of the rule because the necessary condition is that Q sings fourth. Assuming a one-to-one relationship, you couldn't have a $P_4 \longrightarrow Q_4$ outcome.

The above is an alternate presentation of the original idea, and one that for many people crystallizes the essence of what is occurring in an easy-to-digest form. Of course, that alternate diagram works great in this instance (although a bit unwieldy), but could cause issues in other types of games such as Grouping or Pattern games (and, anyway, the test makers don't do us the favor of presenting rules in this fashion because, well, they aren't our friends and they want to keep inherent difficulty).

The key to handling a negative sufficient condition statement or rule is to first realize that a non-occurrence is still an occurrence of sorts; it's just the occurrence of something not happening, if that makes sense. But more importantly, you have to realize that when you see a statement with a negative sufficient condition, that you are dealing with a statement that historically causes confusion (and that the test makers expect to cause confusion, so they focus on it), and that you have to treat with special care. When rules like this appear, you have to mentally underline the rule and commit yourself to remembering how it works and when it applies.

Let's look at another instance of this type of rule, and one that is typically much harder to handle than the prior example:

> If N is not on the committee, then O must be on the committee.
>
> Diagram: $\cancel{N} \longrightarrow O$

In this case, we are looking at a Grouping rule instead of a Linear rule (as in the prior example). Linear-based presentations of this type of rule tend to be easier to handle, probably because it's easy to see each of the numbered spaces and visualize the variable going elsewhere. In Grouping games, the setup is usually different, and when a variable isn't in a group, it's simply Out, and not assigned to some other specific space. That's obviously a bit more abstract, and thus harder to grasp. In addition, the necessary condition variables in Grouping games are usually affected by being forced into a general group instead of an exact spot. So, there's a specific result, but it isn't typically as exact a result as you see when these rules appear in Linear games.

The test makers also tend to focus on the grouping aspect of how the variables interact, and this is often very tricky. Let's look at our rule again, and consider what happens if this game is built around a five person committee that is being selected. Here's the rule again, for references purposes:

If N is not on the committee, then O must be on the committee.

Diagram: $\cancel{N} \longrightarrow O$

In this scenario, when N is not on the committee (Out), then automatically we know that O is on the committee (In). This seems straightforward enough, but let's consider the contrapositive:

Contrapositive Diagram: $\cancel{O} \longrightarrow N$

Under the contrapositive, when O is not on the committee (Out), then automatically we know that N is on the committee (In).

When the rule and its contrapositive are considered together, you can see that if one of the two is not on the committee, then the other *must* be on the committee. This results in the often-missed inference that *either N or O must always be on the committee* (which can then be added to your main diagram). This occurs because there's no chance of them both being absent—if one is absent, the rule specifies that the other *has* to be there. Operationally then, when a rule like the one above appears, you can automatically reserve a space in the group for at least one of the two variables.

This last inference also brings up a second point of confusion, which centers on whether both variables could be present on the committee. Considered in isolation, the rule above does not preclude that possibility, and so the answer is that both could be on the committee. Let's look at it more closely:

> The rule contains a negative sufficient condition that N is not on the committee. When that condition is met, then O must be on the committee. So, if N and O were both on the committee, would that be an issue? No, because if N is on the committee, then the $\cancel{N} \longrightarrow$ O rule is *not* active. In other words, the condition is $\cancel{N}$, and so N occurring is different and doesn't force the rule into play.

Alternatively, again using the $\cancel{N} \longrightarrow O$ rule, consider what would occur if it was initially known that O is in fact on the committee. That would meet the necessary condition in the rule, and thus allow the sufficient to happen ($\cancel{N}$) or not happen (N). In other words, it would be possible for both N and O to be on the committee, regardless of whether you start your analysis with N present or O present.

This last part can seem counter-intuitive if you haven't looked at rules like this before, and it explains why you often see students get hurt by these rules in Grouping games (it's so predictable that when rules like this appear on real LSATs, the miss rate on those games rises significantly, and almost always students see those games as above average in difficulty, if not extremely difficult).

The Double-Not Arrow and Negative Sufficient Conditions

The use of double-not arrows in conjunction with single arrow statements that contain a negated condition is pretty standard. It most frequently appears in Logic Games, but also occasionally in Logical Reasoning Formal Logic questions.

As stated in the *PowerScore LSAT Logical Reasoning Bible*, double-not arrows ($\longleftarrow\!\!\!+\!\!\!\longrightarrow$) are "introduced by conditional statements where exactly one of the terms is negative, or by statements using words such as "no" and "none" that imply the two variables cannot 'go together.' " For example:

If B is selected, then C is not selected.

Diagram: $B \longrightarrow \cancel{C}$

In this case, the diagram above represents the statement as given. However, from the contrapositive, we also know that $C \longrightarrow \cancel{B}$, and combining the two statements yields the inference that B and C can never be selected together. Because writing both of those statements out separately can be time-consuming, they can be combined using the double-not arrow, which results in the following diagram:

Diagram: $B \longleftarrow\!\!\!+\!\!\!\longrightarrow C$

Essentially, then, the double-not arrow means that the variables on each end cannot be selected together in that form (thus, in the diagram above, "B" and "C" cannot both be selected. Under the rule, only "B and C" is ruled out above, and any other combination is acceptable; so, "B and $\cancel{C}$," "$\cancel{B}$ and C," and "$\cancel{B}$ and $\cancel{C}$" would each be acceptable outcomes under that rule.
Here's another example that leads to the same diagram and set of possible outcomes:

No Bs are Cs.

Diagram: $B \leftarrow\!\!\!+\!\!\!\rightarrow C$

Ultimately, when the sufficient condition is positive and the necessary condition is negative (as is the case in $B \longrightarrow \cancel{C}$), the result is a double-not arrow relationship like the one above, where both terms are positive and the negative is contained in the symbol itself. But, this result changes when the sufficient condition is negative and the necessary condition is positive, so let's examine what happens there.

Let's start with a simple single arrow conditional statement:

If B is *not* selected, then C is selected.

Diagram: $\cancel{B} \longrightarrow C$

In this case, the diagram above represents the statement as given. However, from the contrapositive, we also know that $\cancel{C} \longrightarrow B$, and combining the two statements yields the inference that when one is *not* selected, the other *must* be selected. In other words, one of B or C must always be selected and both can never be absent. This is a tough concept, and if you choose to use the double-not arrow to represent this relationship, it appears as follows:

Diagram: $\cancel{B} \leftarrow\!\!\!+\!\!\!\rightarrow \cancel{C}$

This diagram means that the variables on each end cannot be selected together in that form (thus, in the diagram above, "$\cancel{B}$" and "$\cancel{C}$" cannot both occur. But, only "$\cancel{B}$ and $\cancel{C}$" is ruled out above, and any other combination is acceptable; so, "B and $\cancel{C}$," "$\cancel{B}$ and C," and "B and C" would each be acceptable under that rule.

The above usage can be a bit confusing, and so with statements that have a negative sufficient condition our advice is that you should only use the double-not arrow if you are *very* comfortable with it. If you aren't, simply write out the statement and its contrapositive separately, keep in mind the impact that the negative sufficient condition has, and move on from there.

In summary, negative sufficient conditions are tricky, because they revolve around the idea of something not occurring being sufficient. That "absence of occurrence" can be hard to grasp, so when you see statements like this, make sure you carefully track what's occurring, especially in Logic Games where negative sufficient condition rules always play a big rule when they appear.

Double-Not Arrow Summary

Both variables are positive in the diagram:

Statement: If A is selected, then B is not selected.

Diagram: $A \longrightarrow \cancel{B}$

Super Diagram: $A \longleftarrow\!\!\!+\!\!\!\longrightarrow B$

Impossible:

1. Both A and B are selected. (A and B)

Possible:

1. A is selected and B is not selected. (A and $\cancel{B}$)
2. A is not selected and B is selected. (A and $\cancel{B}$)
3. Both A and B are not selected. ($\cancel{A}$ and $\cancel{B}$)

Both variables are negative in the diagram:

Statement: If A is not selected, then B is selected.

Diagram: $\cancel{A} \longrightarrow B$

Diagram: $\cancel{A} \longleftarrow\!\!\!+\!\!\!\longrightarrow \cancel{B}$

Impossible:

1. Both A and B are not selected. ($\cancel{A}$ and $\cancel{B}$)

Possible:

1. A is selected and B is not selected. (A and $\cancel{B}$)
2. A is not selected and B is selected. ($\cancel{A}$ and B)
3. Both A and B are selected. (A and B)